Parametric Study of Statistical Bias in Laser Doppler Velocimetry

Richard D. Gould*
North Carolina State University, Raleigh, North Carolina
and
Warren H. Stevenson† and H. Doyle Thompson‡
Purdue University, West Lafayette, Indiana

Introduction

VARIOUS methods have been proposed for post facto correction of laser Doppler velocimeter (LDV) data to reduce velocity bias error. These include the one- and two-dimensional corrections of McLaughlin and Tiederman,[1] the time between validated data method of Barnett and Bentley,[2] and the particle residence time method of Hoesel and Rodi.[3] Of the these three methods, only the two-dimensional McLaughlin and Tiederman (MT) and the time between data (TBD) methods are considered here. The one-dimensional MT method is clearly limited to low turbulence levels, as pointed out by the original authors. Residence time correction is very difficult to implement in general practice, since the total length of each Doppler burst must be recorded. When frequency shifting is used—as will always be the case at high turbulence levels—most signal processors will exhibit fringe count overflows that make accurate residence time determinations impossible.

Experimental sampling techniques are available that eliminate velocity bias during data acquisition,[4,5] but these techniques require high seeding densities that may not be achievable in some flows. Therefore, a determination of the relative utility of different analytical correction methods applicable at low particle density would be very useful.

Analytical studies[6,7] have generally assumed that velocity bias is dependent upon turbulence intensity and one or more characteristic time scales, i.e., the time between validated signals, time between data samples, and integral turbulence time scale. In the present study, these parameters are varied independently in an effort to quantify the biasing effect.

Experimental Procedure

Simultaneous axial and a radial velocity measurements were made, using a two-color two-component LDV system with frequency shift[8] at four radial locations in a plane two step heights downstream of the step in a sudden expansion flowfield. Seed density was controlled to give the desired coincident data validation rate (DVR) when the processors were free running. The data sampling process itself was controlled by waiting fixed time periods after each velocity validation before obtaining the next measurement. Four nominal fixed waiting time sampling rates were chosen as follows: free running, sampling at one-half the DVR, sampling at one-tenth the DVR, and sampling at 100 Hz. Measurements composed of 6400 individual velocity realizations for each velocity channel were made at each sample rate and seed density condition.

In addition to the standard (unweighted) ensemble average statistics, the two correction methods mentioned earlier were applied. The mean velocity estimate for all cases is given by

$$\overline{U} = \sum_{i=1}^{N} \omega_i u_i \Big/ \sum_{i=1}^{N} \omega_i \tag{1}$$

and the estimate for the variance is given by

$$\overline{u'u'} = \sum_{i=1}^{N} \omega_i (u_i - \overline{U})^2 \Big/ \sum_{i=1}^{N} \omega_i \tag{2}$$

where $\omega_i = 1$ for the standard statistic (unweighted), $\omega_i = 1/(u_i^2 + v_1^2)^{1/2}$ for the two-dimensional MT correction, and $\omega_i = \Delta t_i$ for the TBD correction.

A discrete autocorrelation estimate was made using the method described by Jones[9] and Mayo et al.[10] to give information about the turbulent time scales. The lag time axis was divided into 1000 bins of equal width, $\Delta\tau = 50$ μs, and the lag products for all points up to the maximum lag time, $\tau_{max} = 0.05$ s, were accumulated in appropriate bins. The average of all the autoproducts in each bin was assumed to be the discrete autocorrelation function for the bin. All autocorrelation estimates were made using four data sets (6400 measurements per set) obtained when the DVR = 20,000 Hz at free running conditions. The Eulerian integral (macro) time scale τ_E was estimated by numerically integrating the normalized discrete autocorrelation. The Eulerian microscale then was found by curve fitting the normalized discrete autocorrelation to the exponential form, $\exp(-ct^2)$, near $t = 0$. This technique, described in Ref. 11, gives the Eulerian micro or dissipation time scale as $\tau_\lambda = 1/c$. The corresponding Eulerian macro and microfrequencies are given by $f_E = 1/\tau_E$ and $f_\lambda = 1/\tau_\lambda$, respectively.

Experimental Results and Conclusions

Figures 1 and 2 show the normalized mean velocity bias error for various fixed waiting time sample rates and correction schemes as a function of the coincident data validation rate at 33 and 97% turbulence intensity, respectively. (Measurements at 15 and 63% turbulence intensity, omitted for brevity, gave results consistent with these.) Bias error is calculated by assuming that the measurements at an effective sampling rate of 100 Hz with DVR = 20,000 Hz gives the true unbiased velocity $\overline{U}_{un}$.[4,5] The upper portion of each figure compares the standard unweighted free running statistic with the two-dimensional MT and the TBD corrected statistics. The lower portion of each figure shows the effect of changing the sampling rate.

A number of interesting conclusions can be drawn from the results in Figs. 1 and 2. These are discussed below.

1) Neither of the post facto correction methods employed is completely adequate. The two-dimensional correction appears to be valid for turbulence intensities up to about 35% but overcorrects at higher turbulence levels. The TBD correction method gave mean velocities that were inconsistent and often equaled the uncorrected results. Edwards and Baratuci[12] have suggested that a conditional interarrival time (i.e., TBD on the condition that the velocity lies between V_i and $V_i + \Delta V$) and not simply the TBD is the correct weighting. Chin and Lightman[13] showed that such a weighting shifts the probability distribution in the correct direction.

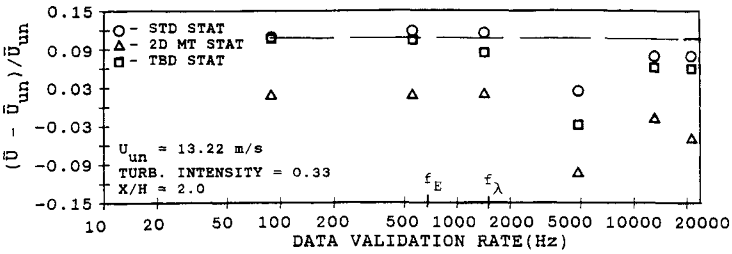
2) The mean velocity bias error was found to be nearly independent of DVR when data were sampled using a free running processor. This nearly constant error was $(\overline{U} - \overline{U}_{un})/\overline{U}_{un} \approx 1/3\,(u'/\overline{U})$ for all turbulence levels. A linear variation of bias with turbulence intensity contradicts the square law dependence proposed theoretically by others.[2,6] However, such linear behavior of bias error for turbulence intensities exceeding 40% was also observed by Petrie et al.[14] in a recent study. They used the two-dimensional MT correction on their biased data that, in the present study, led to an overcorrection by a factor of two at 97% turbulence intensity. If

Received April 22, 1987; revision received June 9, 1988.

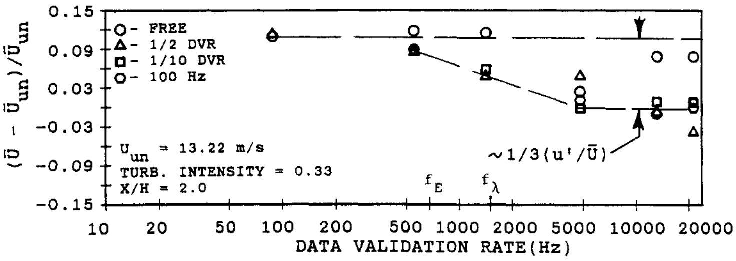
*Assistant Professor, Mechanical and Aerospace Engineering Department.

†Professor, School of Mechanical Engineering.

‡Professor, School of Mechanical Engineering. Associate Fellow AIAA.

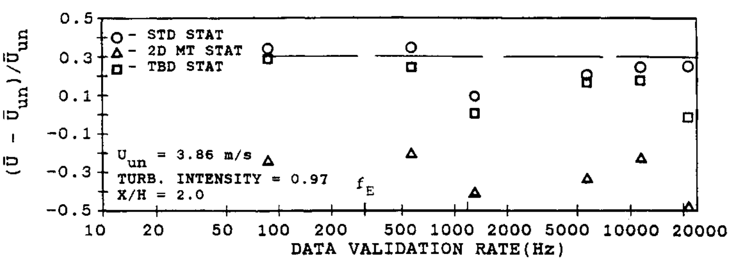


a) Effect of applying two-dimensional MT and TBD correction schemes to the standard (ensemble-averaged) velocity data

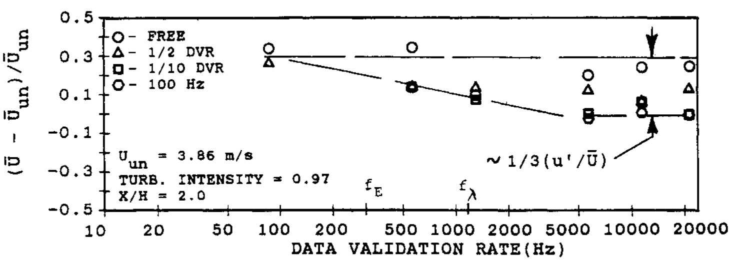


b) Effect of sampling rate on velocity bias

Fig. 1 Mean velocity bias error vs data validation rate (TI = 33%).



a) Effect of applying two-dimensional MT and TBD correction schemes to the standard (ensemble-averaged) velocity rate



b) Effect of sampling rate on velocity bias

Fig. 2 Mean velocity bias error vs data validation data (TI = 97%).

the same is true in their case, the bias in their measurements agrees closely with the empirical relationship found here.

3) Another conclusion to be drawn from the experimental results is that the DVR must be at least three times greater than the turbulent microscale frequency to obtain bias-free results with equal time sampling. Below this seeding limit, the results will be biased regardless of the sampling process. The sloping lines that become horizontal at DVR $> 3f_\lambda$ in Figs. 1 and 2 indicate this trend.

4) The results show that fixed waiting time sampling will give nearly unbiased results (provided DVR $> 3f_\lambda$) as long as the effective sampling rate is less than 1/10 DVR. Even at ½ DVR, the bias error will be reduced.

Further work is needed to completely resolve the velocity bias issue, but it is clear that common analytical correction methods are inadequate and can give inconsistent or erroneous results. Bias can be minimized experimentally by approximate equal time sampling at rates that are low compared to the free running data validation rate, provided the DVR exceeds the turbulent microscale frequency by a factor of three or more. The possibility exists that the simple empirical correction factor described above can be used when such experimental conditions cannot be met. While additional confirmation is required, the present results suggest that this performs well at turbulence levels up to 100%. Finally, it should be noted that the results of this study are in essential agreement with conclusions of the American Society of Mechanical Engineers (ASME) panel on the bias problem as reported in Ref. 15.

Acknowledgments

This investigation was supported by NASA Lewis Research Center under Grant NAG3-503.

References

[1]McLaughlin, D. K. and Tiederman, W. G., "Bias Correction for Individual Realization Laser Anemometry Measurements in Turbulent Flows," *Physics of Fluids,* Vol. 16, No. 12, 1973, pp. 2082–2088.

[2]Barnett, D. and Bentley, H., "Statistical Bias of Individual Realization Laser Velocimeters," *Proceedings of the Second International Workshop on Laser Velocimetry,* Purdue Univ., West Lafayette, IN, 1974, p. 428.

[3]Hoesel, W. and Rodi, W., "New Biasing Elimination Method for Laser Doppler Velocimeter Counter Processing," *Review of Scientific Instruments,* Vol. 48, No. 7, 1977, pp. 910–919.

[4]Stevenson, W. H., Thompson, H. D., and Roesler, T. C., "Direct Measurement of Laser Velocimeter Bias Errors in a Turbulent Flow," *AIAA Journal,* Vol. 20, Dec. 1982, pp. 1720–1723.

[5]Johnson, D. A., Modarress, D., and Owen, F. K., "An Experimental Verification of Laser-Velocimeter Sampling Bias Correction," *Proceedings of Symposium on Engineering Applications of Laser Velocimetry,* American Society of Mechanical Engineers, New York, 1982, pp. 153–162.

[6]Erdmann, J. C. and Tropea, C. D., "Statistical Bias of the Velocity Distribution Function in Laser Anemometry," *Proceedings of First International Symposium on Applications of Laser Anemometry to Fluid Mechanics,* Instituto Superior Technico, Lisbon, Portugal, July 1982, Paper 16.2.

[7]Edwards, R. V. and Meyers, J. F., "An Overview of Particle Sampling Bias," *Proceedings of Second International Symposium on Applications of Laser Anemometry to Fluid Mechanics,* Instituto Superior Technico, Lisbon, Portugal, July 1984, Paper 2.1.

[8]Gould, R. D., Stevenson, W. H., and Thompson, H. D., "Turbulence Characteristics of an Axisymmetric Reacting Flow," NASA CR-4110, Feb. 1988.

[9]Jones, R. J., "Aliasing with Unequally Spaced Observations," *Journal of Applied Meteorology,* Vol. 11, No. 2, 1972, pp. 245–254.

[10]Mayo, W. T., Shay, M. T., and Ritter, S., "The Development of New Digital Data Processing Techniques for Turbulence Measurements with Laser Velocimetry," Final Report AEDC-TR-74-53, Aug. 1974.

[11]Hinze, J. O., *Turbulence,* 2nd ed., McGraw-Hill, New York, 1975.

[12]Edwards, R. V. and Baratuci, W., "Simulation of Particle Measurement Statistics for Laser Anemometers," *Proceedings of Ninth Symposium on Turbulence,* Univ. of Missouri—Rolla, Rolla, MO, 1984, Paper 35.

[13]Chen, T. H. and Lightman, A. J., "Effects of Particle Arrival Statistics on Laser Anemometer Measurements," *Proceedings of International Symposium on Laser Anemometry,* American Society of Mechanical Engineers, New York, 1985, pp. 231–234.

[14]Petrie, H. L., Samimy, M., and Addy, A. L., "Laser Doppler Velocity Bias in Separated Turbulent Flows," *Experiments in Fluids,* Vol. 6, 1988, pp. 80–88.

[15]Edwards, R. V. (ed.), "Report of the Special Panel on Statistical Particle Bias Problems in Laser Anemometry," *Journal of Fluids Engineering,* Vol. 109, June 1987, pp. 89–93.

Noise Bias in Various Formulations of Ibrahim's Time Domain Technique

Joseph J. Hollkamp* and Stephen M. Batill†

University of Notre Dame, Notre Dame, Indiana

Introduction

IBRAHIM's Time Domain Technique (ITD) estimates natural frequencies, damping factors, and mode shapes using sampled, free response data in the time domain. In the ITD, either the position, velocity, or acceleration is measured at n locations (for a system with n degrees of freedom) at discrete time intervals. These measurements are used to form the coefficient matrix of an eigenvalue problem. The eigenvalues and eigenvectors of this matrix can be directly related to the natural frequencies, damping factors, and mode shapes. If there is no noise in the measurements, then only $2n$ discrete time samples are needed to form a coefficient matrix to precisely determine the vibration parameters. More data is required when noise is present in order to estimate the coefficient matrix in a least squares (LS) sense.[1] However, it has been recognized that often the LS algorithms result in biased estimation, and therefore alternate methods such as Maximum Likelihood and Instrumental Variables have been proposed to eliminate the bias problems.[2] Reference 3 presents a double least squares (DLS) procedure in an effort to reduce the bias in the LS ITD technique. This study compares the results from the DLS and the LS techniques with an unbiased Instrumental Variable (IV) approach. The analytical expressions for the bias terms in the LS and the DLS ITD methods are provided.

Received Dec. 21, 1987; revision received April 28, 1988.

*Graduate Research Assistant, Department of Aerospace and Mechanical Engineering. Student Member AIAA.

†Associate Professor, Department of Aerospace and Mechanical Engineering. Associate Fellow AIAA.

ITD Theory

The equations of motion for the free response of a discrete, multiple-degree-of-freedom, damped system can be written as

$$M\frac{d^2\boldsymbol{x}}{dt^2}+C\frac{d\boldsymbol{x}}{dt}+K\boldsymbol{x}=0 \tag{1}$$

where $\boldsymbol{x}$ is a vector of responses at n assumed degrees of freedom. The solutions of these equations are of the form $\boldsymbol{x}(t)=\boldsymbol{p}\exp(\lambda t)$. Substitution into Eq. (1) yields the eigenvalue problem

$$[\lambda^2M+\lambda C+K]\boldsymbol{p}=0 \tag{2}$$

There exist $2n$ eigenvalues, λ. Here, $\boldsymbol{p}$ represents the complex modes of vibration. The system response is found by the summation, and the mode shape scaling depends upon the initial conditions

$$\boldsymbol{x}(t)=\sum_{j=1}^{2n}\boldsymbol{p}_j\exp(\lambda_j t) \tag{3}$$

Application of the ITD technique[1] for "noise-free" data results in the expression

$$\Phi'\Phi^{-1}\Psi_i=\exp(\lambda_i\Delta t)\Psi_i \tag{4}$$

where Φ and Φ' are developed from $2n$ discrete time values of $\boldsymbol{x}(t)$. The complex eigenvalues and eigenvectors of $\Phi'\Phi^{-1}$ can be used to describe completely the vibration parameters of the system.

Measurements with Noise

Experimental data usually is contaminated with noise, and therefore the eigenvalue problem must be modified to deal with noise. The modification proposed in Ref. 1 uses an LS algorithm and, though it was recognized that the LS estimates were biased,[3] the analytical expression for the bias was not provided. The bias led to a DLS approach[3] in an effort to reduce the influence of the noise on the parameter estimates.

Consider the case where the response data is corrupted with noise. The measured response is

$$\boldsymbol{x}_m(t)=\sum_{j=1}^{2n}\boldsymbol{p}_j\exp(\lambda_j t)+\boldsymbol{n}(t) \tag{5}$$

where $\boldsymbol{n}(t)$ is a stochastic vector comprised of the noise process at each measurement location. An overdetermined set of equations is needed for the LS algorithm. Measurements at r